① Practical-1 & 2:-

calculate f(1.151), f(1.156), f(1.148), f(1.183), f(1.179), & f(1.187) from the following table:

x	f(x)
1.150	0.91276394026
1.155	0.91479495943
1.160	0.91680310877
1.165	0.91878833808
1.170	0.92075059774
1.175	0.92268983867
1.180	0.92460601241
1.185	0.92649907104

Difference table -

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.150	91276394026					
		203101917				
1.155	91479495943		-2286983			
		200814934		-5020		
1.160	91680310877		-2292003		58	
		198522931		-4962		-4
1.165	91878833808		-2296965		54	
		196225966		-4908		8
1.170	92075059774		-2301873		62	
		193924093		-4846		-8
1.175	92268983867		-2306719		54	
		191617374		-4792		↓ Noise level
1.180	92460601241		-2311511			
		189305863				
1.185	92649907104					

$f(1.151)$:-

$x = 1.151$, $x_0 = 1.150$, $h = 0.005$, $u = \frac{x - x_0}{h} = 0.2$

The Newton's forward interpolation formula is

$$f(x) \simeq y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

Coefficient	multiplier	positive term	negative term
1	0.91276394026	0.9127639402600	
0.2	0.00203101917	0.0004062038340	
−0.08	−0.00002286983	0.0000018295864	
0.048	−0.0000000 5020		0.00000000024096
−0.0336	0.00000000058		0.00000000000195
		0.9131719736804	0.00000000024291

Difference :-

$$\begin{array}{r} 0.9131719736804 \\ -\ 0.0000000024291 \\ \hline 0.9131719712513 \end{array}$$

$\therefore$ $f(1.151) \simeq 0.913171971 25$

$f(1.156)$:-

$x = 1.156$, $x_0 = 1.155$, $h = 0.005$, $u = \frac{x - x_0}{h} = 0.2$

$$f(x) \simeq y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

coefficient	multiplier	positive term	negative term
1	0.9147949 5943	0.9147949 5943	
0.2	0.0020081 4934	0.00040162 9868 0	
−0.08	−0.0000 2292003	0.0000018336024	
0.048	−0.0000000 4962		0.0000000023818
−0.0336	0.00000000054		0.000000000181
		0.915198 4229004	0.0000000023999

Difference:-

$$\begin{array}{r} 0.915198422 9004 \\ -\ 0.000000002 3999 \\ \hline 0.915198420 5005 \end{array}$$

$f(1.156) \simeq 0.9151984205 0$

$f(1.148)$:-

$x = 1.148$, $x_0 = 1.150$, $h = 0.005$, $u = \frac{x - x_0}{h} = -0.4$

$$f(x) \approx y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

coefficient	multiplier	positive term	negative term
1	0.91276394026	0.9127639402600	
−0.4	0.00203101917		0.0008124076680
0.28	−0.000002286983		0.00000064035524
−0.224	−0.00000005020	0.00000000112448	
0.1904	0.000000000058	0.00000000001104	
		0.9127639516152	0.0008188112204

Difference :-

$$\begin{array}{r} 0.9127639516152 \\ 0.0008188112204 \\ \hline 0.9119451403948 \end{array}$$

$f(1.148) \simeq 0.911945140 39$

f(1.183):-

$x = 1.183$, $x_n = 1.185$, $h = 0.005$, $v = \frac{x - x_n}{h} = -0.4$

The Newton's backward interpolation formula is

$$f(x) \simeq y_n + \frac{v}{1!}\Delta y_{n-1} + \frac{v(v+1)}{2!}\Delta^2 y_{n-2} + \frac{v(v+1)(v+2)}{3!}\Delta^3 y_{n-3}$$

$$+ \frac{v(v+1)(v+2)(v+3)}{4!}\Delta^4 y_{n-4}$$

Coefficient	multiplier	positive term	negative term
1	0.92649907104	0.9264990710400	
-0.4	0.00189305863		0.0007572234520
-0.12	-0.00002311511	0.0000027738132	
-0.064	-0.00000004792	0.000000003066 9	
-0.0416	0.00000000054		0.000000000022 5
		0.9265018479201	0.0007572234745

Difference:

$$\begin{array}{r} 0.9265018479201 \\ 0.0007572234745 \\ \hline 0.9257446244456 \end{array}$$

$f(1.183) \simeq 0.925744624445$

f(1.179):-

$x = 1.179$, $x_n = 1.180$, $h = 0.005$, $v = \frac{x - x_n}{h} = -0.2$

$$f(x) \simeq y_n + \frac{v}{1!} \Delta y_{n-1} + \frac{v(v+1)}{2!} \Delta^2 y_{n-2} + \frac{v(v+1)(v+2)}{3!} \Delta^3 y_{n-3}$$

$$+ \frac{v(v+1)(v+2)(v+3)}{4!} \Delta^4 y_{n-4}$$

Coefficient	multiplier	positive term	negative term
1	0.92460601241	0.92460601241200	
-0.2	0.00191617374		0.0003832347480
-0.08	-0.00002306719	0.0000018453752	
-0.048	-0.0000000 4846	0.00000000023261	
-0.0336	0.00000000062		0.000000000000208
		0.924607860113	0.000383 2347688

Difference :

0.92460786011 13
0.00038323476 88
0.92422462534 25

$f(1.179) \simeq 0.9242246253 4$

f(1.187):-

$x = 1.187$, $x_n = 1.185$, $h = 0.005$, $v = \frac{x - x_n}{h} = 0.4$

$$f(x) \simeq y_n + \frac{v}{1!} \Delta y_{n-1} + \frac{v(v+1)}{2!} \Delta^2 y_{n-2} + \frac{v(v+1)(v+2)}{3!} \Delta^3 y_{n-3}$$

$$+ \frac{v(v+1)(v+2)(v+3)}{4!} \Delta^4 y_{n-4}$$

Coefficient	multiplier	positive term	negative term
1	0.92649907104	0.92649907104 00	
0.4	0.00189305863	0.000757223452 0	
0.28	−0.0000 2311511		0.0000064722308
0.224	−0.00000004792		0.0000000107341
0.1904	0.00000000054	0.000000000102 8	
		0.927256294594 8	0.0000064829649

Difference:-

$$\begin{array}{r} 0.9272562945948 \\ -\ 0.0000064829649 \\ \hline 0.9272498116299 \end{array}$$

$f(1.187) \simeq 0.92724981163$

practical - 3 :-

Calculate the value of f(1.169), f(1.164), f(1.171) from the table given in previous problem.

Soln :- Since the ~~ta~~ difference table ends with even order differences we apply Bessel's formula.

Here $x = 1.164$, $x_0 = 1.160$, $h = 0.005$

$$u = \frac{x - x_0}{h} = 0.8, \quad v = u - \frac{1}{2} = 0.3$$

$$f(x) \simeq \frac{y_0 + y_1}{2} + v\Delta y_0 + \frac{v^2 - \frac{1}{4}}{2!} \cdot \frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}$$

$$+ \frac{v(v^2 - \frac{1}{4})}{3!} \Delta^3 y_{-1} + \frac{(v^2 - \frac{1}{4})(v^2 - \frac{9}{4})}{4!} \cdot \frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}$$

coefficient	multiplier	positive term	negative term
1	0.9177957234250	0.9177957234250	
0.3	0.001985229310	0.0005955687930	
−0.08	−0.0000229448840	0.0000018355872	
−0.008	−0.0000000049620	0.0000000003970	
0.0144	0.0000000000560	0.0000000000081	
		0.9183931282103	

$f(1.164) \simeq 0.918393128221$

$f(1.169)$:-

$x = 1.169, \quad x_0 = 1.165, \quad h = 0.005, \quad u = \frac{x - x_0}{h} = 0.8,$

$v = u - \frac{1}{2} = 0.3$

$$f(x) \simeq \frac{y_0 + y_1}{2} + v\Delta y_0 + \frac{v^2 - \frac{1}{4}}{2!} \cdot \frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}$$

$$+ \frac{v\left(v^2 - \frac{1}{4}\right)}{3!} \cdot \Delta^3 y_{-1} + \frac{\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)}{4!} \cdot \frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}$$

coefficient	multiplier	positive term	negative term
1	0.919769.46791	0.9197694679100	
0.3	0.0019622 5966	0.0005886778980	
−0.08	−0.00002299419	0.0000018395352	
−0.008	−0.00000004908	0.000000000 3926	
0.0144	0.00000000058	0.0000000000084	
		0.9203599857442	

$f(1.169) \simeq 0.92035998574$

$f(1.171)$:—

$x = 1.171, \quad x_0 = 1.170, \quad h = 0.005, \quad u = \frac{x - x_0}{h} = 0.2,$

$v = u - \frac{1}{2} = -0.3$

$$f(x) \simeq \frac{y_0 + y_1}{2} + v\Delta y_0 + \frac{v^2 - \frac{1}{4}}{2!} \cdot \frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}$$

$$+ \frac{v\left(v^2 - \frac{1}{4}\right)}{3!} \cdot \Delta^3 y_{-1} + \frac{\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)}{4!} \cdot \frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}$$

coefficient	multiplier	positive term	negative term
1	0.92172021825	0.9217202182500	
-0.3	0.00193924093		0.000581772279 0
-0.08	-0.00002304296	0.0000018434368	
0.008	-0.00000004846		0.00000000038 77
0.0144	0.00000000058	0.00000000000084	
		→ 0.92172206169 52	0.00058177266 67

Difference :

$$\begin{array}{r} 0.92172206169 52 \\ 0.00058177266 67 \\ \hline 0.92114028902 85 \end{array}$$

$f(1.171) \simeq 0.92114028903$

practical-4 :-

Calculate $f(1.169)$ from the following table by Stirling's interpolation.

x	$f(x)$
1.150	0.9127639403
1.155	0.9147949594
1.160	0.9168031088
1.165	0.9187883381
1.170	0.9207505977
1.175	0.9226898387
1.180	0.9246060124
1.185	0.9264990710

Difference table

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.150	9127639403					
		20310191				
1.155	9147949594		-228697			
		20081494		-504		
1.160	9168031088		-229201		8	
		19852293		-496		-1
1.165	9187883381		-229697		7	
		19622596		-489		-5
1.170	9207505977		-230186		2	
		19392410		-487		7
1.175	9226898387		-230673		9	
		19161737		-478		
1.180	9246060124		-231151			
		18930586				
1.185	9264990710					

↓ Noise level

$f(1.169)$:-

$x = 1.169$, $x_0 = 1.165$, $h = 0.005$, $u = \frac{x - x_0}{h} = 0.8$

$$f(x) \simeq y_0 + \frac{u}{1!} \cdot \frac{\Delta y_0 + \Delta y_{-1}}{2} + \frac{u^2}{2!} \Delta^2 y_{-1}$$

$$+ \frac{u(u^2-1^2)}{3!} \cdot \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \frac{u^2(u^2-1^2)}{4!} \Delta^4 y_{-2}$$

coefficient	multiplier	positive term	negative term
1	0.91878833810	0.918788338100	
0.8	0.0019737444 5	0.001578995560	
0.32	−0.0000229697 0		0.000007350304
−0.048	−0.0000000492 5	0.0000000023 64	
−0.0096	0.0000000000170		0.0000000000167
		0.920367336024	0.000007350311

Difference :

$$\begin{array}{r} 0.920367336024 \\ 0.000007350311 \\ \hline 0.920359985713 \end{array}$$

$f(1.169) \simeq 0.920359985 7$

Practical - 5 :-

Find the value of y for x = 0.151 and x = 0.159 from the following table :

x	y
0.13	0.878095
0.15	0.860708
0.16	0.852144
0.18	0.835270
0.19	0.826959

Soln :- (i) Let $t = \frac{x-\alpha}{\beta}$, $\alpha = 0.16$, $\beta = 0.01$

$\therefore t_0 = \frac{x_0 - \alpha}{\beta} = -3$, $t_1 = \frac{x_1 - \alpha}{\beta} = -1$

$t_2 = \frac{x_2 - \alpha}{\beta} = 0$, $t_3 = \frac{x_3 - \alpha}{\beta} = 2$

$t_4 = \frac{x_4 - \alpha}{\beta} = 3$, $t = \frac{x-\alpha}{\beta} = \frac{0.151 - 0.16}{0.01}$

$= -0.9$

row product →					D_i	y_i	y_i / D_i
2.1	−2	−3	−5	−6	378	0.878095	$2.323002646 \times 10^{-3}$
2	0.1	−1	−3	−4	−2.4	0.860708	$-358.6283333 \times 10^{-3}$
3	1	−0.9	−2	−3	−16.2	0.852144	$-52.60148148 \times 10^{-3}$
5	3	2	−2.9	−1	87	0.835270	$9.600804598 \times 10^{-3}$
6	4	3	1	−3.9	−280.8	0.826959	$-2.945010684 \times 10^{-3}$
							$-402.2510182 \times 10^{-3}$

$$\sum_{i=0}^{4} y_i/D_i = -402.2510182 \times 10^{-3}$$

$$w(t) = -2.13759$$

$$\therefore \ f(0.151) \approx w(t) \sum_{i=0}^{4} y_i/D_i = 0.859848$$

(ii) Let $t = \frac{x-\alpha}{\beta}$, $\alpha = 0.16$, $\beta = 0.01$

$t_0 = \frac{x_0 - \alpha}{\beta} = -3$, $t_1 = \frac{x_1 - \alpha}{\beta} = -1$

$t_2 = \frac{x_2 - \alpha}{\beta} = 0$, $t_3 = \frac{x_3 - \alpha}{\beta} = 2$

$t_4 = \frac{x_4 - \alpha}{\beta} = 3$ $t = \frac{x-\alpha}{\beta} = \frac{0.159 - 0.16}{0.01}$

$= -0.1$

row product →					D_i	y_i	y_i/D_i
2.9	−2	−3	−5	−6	522	0.878095	$1.68217433 \times 10^{-3}$
2	0.9	−1	−3	−4	−21.6	0.860708	$-39.84759259 \times 10^{-3}$
3	1	−0.1	−2	−3	−1.8	0.852144	$-473.4133333 \times 10^{-3}$
5	3	2	−2.1	−1	63	0.835270	$13.25825397 \times 10^{-3}$
6	4	3	1	−3.1	−223.2	0.826959	$-3.705013441 \times 10^{-3}$
							−0.502025511

$w(t) = -1.69911$

$$\therefore f(0.159) \approx w(t) \sum_{i=0}^{4} y_i/D_i = 0.852997$$

practical-6 :-

Find the value of y when $x = 0.151$ & $x = 0.159$ from the following table :

x	y
0.13	0.878095
0.15	0.860708
0.16	0.852144
0.18	0.835270
0.19	0.826959

Divided difference table :-

				x	$f(x)$	δ	δ^2	δ^3	δ^4
				0.13	0.878095				
			−0.02			−0.86935			
		−0.03		0.15	0.860708		0.431666666		
	−0.05		−0.01			−0.85640		−0.166666660	
−0.06		−0.03		0.16	0.852144		0.423333333		1.388888917
	−0.04		−0.02			−0.84370		−0.083333325	
		−0.03		0.18	0.835270		0.420000000		
			−0.01			−0.83110			
				0.19	0.826959				

$f(0.159)$:—

$$f(x) \simeq f(x_0) + (x-x_0) f[x_0,x_1] + (x-x_0)(x-x_1) f[x_0,x_1,x_2]$$
$$+ (x-x_0)(x-x_1)(x-x_2) f[x_0,x_1,x_2,x_3]$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3) f[x_0,x_1,x_2,x_3,x_4]$$

Coefficient	multiplier	positive term	negative term
1	0.878095	0.87809500	
0.029	−0.869350		0.02521115
2.61×10^{-4}	0.431666666	0.00011266	
-2.61×10^{-7}	−0.16666666	0.00000004	
5.481×10^{-9}	1.388888917	0.00000001	
		0.87820771	0.02521115

Difference:

$$\begin{array}{r} 0.87820771 \\ -\ 0.02521115 \\ \hline 0.85299656 \end{array}$$

$f(0.159) \simeq 0.8529967$

(ii)

$f(0.151)$:—

$$f(x) \simeq f(x_0) + (x-x_0) f[x_0,x_1] + (x-x_0)(x-x_1) f[x_0,x_1,x_2]$$
$$+ (x-x_0)(x-x_1)(x-x_2) f[x_0,x_1,x_2,x_3]$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3) f[x_0,x_1,x_2,x_3,x_4]$$

coefficient	multiplier	positive term	negative term
1	0.878095	0.87809500	
0.021	-0.869350		0.01825635
0.021×10^{-3}	0.431666666	0.00000906	
-0.189×10^{-6}	-0.16666666	0.00000003	
5.481×10^{-9}	1.388888917	0.00000001	
		0.87810410	0.01825635

Difference: 0.87810410
− 0.01825635
= 0.85984775

$\therefore f(0.151) \simeq 0.859848$

Practical - 7 :-

calculate $f'(1.152)$, $f''(1.150)$, $f''(1.185)$, $f'(1.183)$ from the table given in practical - 1.

Soln :- $f'(1.152)$:-

$x = 1.152$, $x_0 = 1.150$, $h = 0.005$, $u = \frac{x - x_0}{h} = 0.4$

$$f'(x) \simeq \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0\right]$$

coefficient	multiplier	positive term	negative term
1	0.0020310191 7	0.0020310191700	
−0.1	−0.00002286983	0.0000022869830	
$\frac{1}{75}$	−0.0000005020		0.0000000006693
$\frac{11}{1500}$	0.0000000058	0.0000000000043	
		0.0020333061573	0.0000000006693

Difference: 0.0020333061573
0.0000000006693
0.0020333054880

$\therefore f'(1.152) \simeq \frac{1}{0.005} \times (0.0020333054880)$

$= 0.406661097$

▫ $f''(1.150)$:—

$x = 1.150$, $x_0 = 1.150$, $h = 0.005$, $u = \frac{x - x_0}{h} = 0$

$$f''(x) \simeq \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0\right]$$

coefficient	multiplier	positive term	negative term
1	−0.00002286983		0.0000228698300
−1	−0.0000005020	0.0000000502000	
$\frac{11}{12}$	0.0000000058	0.0000000005310	
		0.0000000507310	0.0000228698300

~~Difference: 0.0000000507310~~
~~0.0000228698300~~
~~97718090 10~~

Difference :-

$$\begin{array}{r} 0.00002286\,98300 \\ -0.0000000050\,7310 \\ \hline -0.00002281\,90990 \end{array}$$

$$f''(1.150) \simeq -\frac{1}{(0.005)^2} \times 0.000022819\,0990$$

$$= -0.91276396000$$

■ $f''(1.185)$:-

$x_n = 1.185$, $x = 1.185$, $h = 0.005$, $v = \frac{x - x_n}{h} = 0$

$$f''(x) \simeq \frac{1}{h^2}\left[\Delta^2 y_{n-2} + \Delta^3 y_{n-3} + \frac{11}{12}\Delta^4 y_{n-4}\right]$$

coefficient	multiplier	positive term	negative term
1	−0.0000231151 1		0.00002311511 00
1	−0.00000004792		0.0000000479200
$\frac{11}{12}$	0.0000000054	0.000000004950	
		0.000000004950	0.0000231630300

Difference :

$$\begin{array}{r} 0.0000231630300 \\ -0.0000000004950 \\ \hline -0.0000231625350 \end{array}$$

$$f''(1.185) \simeq \frac{1}{(0.005)^2} \times (-0.0000231625350)$$

$$= -0.9265014$$

Practical :-

Evaluate the following integral with 13 ordinates correct upto 6D by (i) Trapezoidal rule (ii) Simpson-one third rule (iii) Weddles' rule.

$$\int_{10^\circ}^{30^\circ} \frac{dx}{\sqrt{(1-0.4\sin^2 x)(1+\sin^2 x)}}.$$

Soln :- (i)

$$f(x) = \frac{1}{\sqrt{(1-0.4\sin^2 x)(1+\sin^2 x)}}.$$

$$a = 10^\circ = \frac{\pi}{18}, \quad b = 30^\circ = \frac{\pi}{6}, \quad n = 12$$

$$h = \frac{b-a}{n} = \frac{\pi}{108}$$

i	x_i	$f(x_i)$	$f(x_i), i=0,12$	$f(x_i), i=1,2,\ldots,11$
0	$\pi/18$	0.99125189	0.99125189	
1	$7\pi/108$	0.98827629		0.98827629
2	$8\pi/108$	0.98495697		0.98495697
3	$9\pi/108$	0.98133704		0.98133704
4	$10\pi/108$	0.97746189		0.97746189
5	$11\pi/108$	0.97337834		0.97337834
6	$12\pi/108$	0.96913369		0.96913369
7	$13\pi/108$	0.96477494		0.96477494
8	$14\pi/108$	0.96034805		0.96034805
9	$15\pi/108$	0.95589730		0.95589730
10	$16\pi/108$	0.95146475		0.95146475
11	$17\pi/108$	0.94708984		0.94708984
12	$18\pi/108$	0.94280904	0.94280904	
			1.93406093	10.65411910

$$I \simeq \frac{h}{2}\left[f(x_0)+f(x_{12}) + 2\times\{f(x_1)+f(x_2)+\cdots+f(x_{11})\}\right]$$

$$= \frac{\pi}{216}\left[1.93406093 + 2 \times 10.65411910\right]$$

$= 0.338046$ (correct upto 6D)

(11)

i	x_i	$f(x_i)$	$f(x_i)$, $i=0,12$	$f(x_i)$, $i=$odd	$f(x_i)$, $i=$even
0	$\pi/18$	0.99125189	0.99125189		
1	$7\pi/108$	0.98827629		0.98827629	
2	$8\pi/108$	0.98495697			0.98495697
3	$9\pi/108$	0.98133704		0.98133704	
4	$10\pi/108$	0.97746189			0.97746189
5	$11\pi/108$	0.97337834		0.97337834	
6	$12\pi/108$	0.96913369			0.96913369
7	$13\pi/108$	0.96477494		0.96477494	
8	$14\pi/108$	0.96034805			0.96034805
9	$15\pi/108$	0.95589730		0.95589730	
10	$16\pi/108$	0.95146475			0.95146475
11	$17\pi/108$	0.94708984		0.94708984	
12	$18\pi/108$	0.94280904	0.94280904		
			1.93406093	5.81075375	4.84336535

$$I \simeq \frac{h}{3}\left[f(x_0)+f(x_{12})+4\left\{f(x_1)+f(x_3)+\cdots+f(x_{11})\right\}+2\left\{f(x_2)+f(x_4)+\cdots+f(x_{10})\right\}\right]$$

$$=\frac{\pi}{324}\left[1.93406093+4\times 5.81075375+2\times 4.84336535\right]$$

$= 0.338049$ (correct upto 6d)

(iii)

i	x_i	$f(x_i)$	$f(x_i), i=0,12$	$f(x_i), i=1,5,7,11$	$f(x_i), i=2,4,8,10$	$f(x_i), i=3,9$	$f(x_i), i=6$
0	$\pi/18$	0.99125189	0.99125189				
1	$7\pi/108$	0.98827629		0.98827629			
2	$8\pi/108$	0.98495697			0.98495697		
3	$9\pi/108$	0.98133704				0.98133704	
4	$10\pi/108$	0.97746189			0.97746189		
5	$11\pi/108$	0.97337834		0.97337834			
6	$12\pi/108$	0.96913369					0.96913369
7	$13\pi/108$	0.96477494		0.96477494			
8	$14\pi/108$	0.96034805			0.96034805		
9	$15\pi/108$	0.95589730				0.95589730	
10	$16\pi/108$	0.95146475			0.95146475		
11	$17\pi/108$	0.94708984		0.94708984			
12	$18\pi/108$	0.94280904	0.94280904				
			1.93406093	3.87351941	3.87423166	1.93723434	0.96913369

$$I \simeq \frac{3h}{10}\Big[f(x_0)+f(x_{12})+5\{f(x_1)+f(x_5)+f(x_7)+f(x_{11})\} + \{f(x_2)+f(x_4)+f(x_8)+f(x_{10})\}+6\{f(x_3)+f(x_9)\} + 2f(x_6)\Big]$$

$$= \frac{\pi}{360}\Big[1.93406093+5\times 3.87351941+3.87423166 +6\times 1.93723434+2\times 0.96913369\Big]$$

$$= 0.338049 \text{ (correct upto 6D).}$$